Topcolor assisted technicolor models and muon anomalous magnetic moment

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Abstract

We discuss and estimate the contributions of the new particles predicted by topcolor assisted technicolor(TC2) models to the muon anomalous magnetic moment a_μ. Our results show that the contributions of Pseudo Goldstone bosons are very small which can be safely ignored. The main contributions come from the ETC gauge boson x_μ and topcolor gauge boson Z'. If we demand that the mass of Z' is consistent with other experimental constrains, its contributions are smaller than that of x_μ. With reasonable values of the parameters in TC2 models, the observed BNL results for a_μ could be explained.

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A new experimental value of the muon anomalous magnetic moment, $a_\mu = \frac{1}{2}(g-2)_\mu$, was recently reported by BNL[1]. The data seems to include a 2.6 standard deviation from theoretical predictions based on the standard model(SM). This may be an indication of new physics if it is more than a statistical fluctuation. If the deviation comes from the new physics effects, then at 95% CL, δa_μ^{NP} must lie in the range[2]:

$$215 \times 10^{-11} \leq \delta a_\mu^{NP} \leq 637 \times 10^{-11}. \tag{1}$$

Many authors have considered various possibilities of interpreting the deviation in models beyond the SM, such as supersymmetry theories[3, 4], technicolor theories[5], muon with substructure in composite models[6] and models with extra neutral gauge bosons or fermions[7]. In this letter, we estimate the contributions of the new particles (technipions, top-pions, ETC gauge bosons and extra $U(1)_Y$ gauge boson Z') predicted by topcolor assisted technicolor(TC2) models[8] to the muon anomalous magnetic moment a_μ. Our results show that, with reasonable values of the parameters in TC2 models, the deviation could be explained.

To solve the phenomenological difficulties of traditional technicolor(TC) theory, TC2 models[8] were proposed by combing TC interactions with the topcolor interactions for the third generation at the scale of about 1 TeV. In TC2 models, there are two kinds of new gauge bosons: (a) ETC gauge bosons, (b) the topcolor gauge bosons B_μ^A and Z'. Furthermore, this kind of models predict a number of Pseudo Goldstone bosons (PGBs) including the technipions in the TC sector and the top-pions in the topcolor sector. It has been shown[9] that all of these new particles can give corrections to the Z-pole observablities at LEP and SLD, and thus the LEP-SLD precision data could give constrains on the parameters of TC2 models. In this letter, we consider the contributions of these new particles to the muon anomalous magnetic moment a_μ and see whether the recent measured value of a_μ can give constrains on the parameters of TC2 models.

Before numerical estimations, we give the couplings of these new particles to muons. The couplings of PGBs to muons can be written as[8, 10]:

$$\frac{im_\mu}{\sqrt{2}F_T}[\pi_p^0\bar{\mu}\gamma^5\mu + \bar{\nu}_{\mu L}\pi_p^+\mu_R + \bar{\mu}_L\pi_p^-\nu_{\mu R}] + \frac{im_\mu}{\sqrt{2}F_t}(\frac{F_t}{v_w})[\pi_t^0\bar{\mu}\gamma^5\mu + \bar{\nu}_{\mu L}\pi_t^+\mu_R + \bar{\mu}_L\pi_t^-\nu_{\mu R}], \tag{2}$$

where F_T is the decay constant of the technipion π_p, $F_t \simeq 50 GeV$ is the decay constant of the top-pion π_t and $v_w = v/\sqrt{2} = 174 GeV$.

For TC2 models, the underlying interactions, topcolor interactions, are non-universal and therefore do not possess a GIM mechanism[11]. When the non-universal interactions are written in the mass eigen basis, it lead to the flavor changing (FC) vertices of the new gauge bosons, such as $Z'tc$, $Z'\mu e$ and $Z'\mu\tau$[11, 12]. The couplings of Z' to muons , including the $\mu - \tau$ transition, can be written as[8, 12]:

$$-\frac{1}{2}g_1 \tan\theta'(\bar{\mu}_L\gamma^\mu\mu_L + 2\bar{\mu}_R\gamma^\mu\mu_R) - g_1[K_L(\bar{\mu}_L\gamma^\mu\tau_L) + 2K_R(\bar{\mu}_R\gamma^\mu\tau_R)], \tag{3}$$

where g_1 is the $U(1)_Y$ coupling constant at the scale Λ_{TC}, θ' is the mixing angle with $\tan\theta' = g_1/(2\sqrt{\pi K_1})$. K_L and K_R are the flavor mixing factors. In the following estimation, we will assume $|K_L| = |K_R| = V_{23}$, which is the matrix element of the lepton mixing matrix V. In order to explain the atmospheric neutrino results, one needs an almost maximal mixing between ν_μ and ν_τ. Thus, we take a large mixing between the second and third generation leptons, i.e. $V_{23} = 1/\sqrt{2}$.

For TC2 models, the TC interactions play a main role in breaking the electroweak symmetry. The topcolor interactions also make small contributions to the electroweak symmetry breaking, and give rise to the main part of the top quark mass, $(1 - \epsilon)m_t$ with a model dependent parameter $\epsilon \ll 1$. In TC2 models, ETC interactions are still needed to generate the masses of the light quarks and leptons, give masses to PGBs, and contribute a few GeV to m_t, i.e. ϵm_t. Ordinary fermions receive the ETC induced masses via the ETC couplings of technifermions to ordinary fermions. The couplings of the ETC gauge boson x_μ to muons can be written as:

$$g_E(\zeta_\mu \bar{L}_L\gamma^\mu l_L + \zeta_{R\mu}\bar{E}_R\gamma^\mu\mu_R), \tag{4}$$

where $L_L = (N, E)$ and E_R represent technifermions, and $l_L = (\nu_\mu, \mu)_L$, μ_R represent the second generation leptons. Ordinary fermions couple to the techinfermions via ETC interactions with the coupling constant g_E, ζ_μ and $\zeta_{R\mu}$ are the coefficients of the left- and the right-handed couplings.

From eq.2 we can see that the PGBs (π_p and π_t) can give the one-loop corrections to the muon anomalous magnetic moment a_μ via the Feynman diagrams shown in Fig.1. Calculating Fig.1, we can give the corrections of PGBs to a_μ, which can be approximately written as:

$$
\begin{aligned}
\delta a_\mu^{PGB} &= \delta a_\mu^{\pi_p^0} + \delta a_\mu^{\pi_p^\pm} + \delta a_\mu^{\pi_t^0} + \delta a_\mu^{\pi_t^\pm} \\
&\approx \frac{G_F m_\mu^2}{4\pi^2\sqrt{2}} \{ (\frac{m_\mu^2}{m_{\pi_p}^2})(\frac{\upsilon}{F_T})^2 [\ln(\frac{m_{\pi_p}^2}{m_\mu^2}) - \frac{7}{6}] + (\frac{m_\mu^2}{m_{\pi_t}^2})[\ln(\frac{m_{\pi_t}^2}{m_\mu^2}) - \frac{7}{6}] \}.
\end{aligned}
\tag{5}
$$

The contributions of the charged PGB's to a_μ are much smaller than those of the neutral PGB's, which have been neglected in above equation. It has been pointed out that the mass of the lightest technipion π_p^0 is about $100 GeV$[10] and the mass of the top-pion π_t^0 is in the range of $200 \sim 400 GeV$[8, 9]. If we take $F_T = 40 GeV$, which corresponds to the topcolor assisted multiscale technicolor models[9], then we find $\delta a_\mu^{PGB} \leq 1.2\times10^{-12}$. Thus, the corrections of the PGBs to a_μ can be safely neglected.

The one-loop corrections of the extra $U(1)_Y$ gauge boson Z' to the muon anomalous magnetic moment can be divided into two parts: one part comes from the coupling $Z'\mu\bar{\mu}$ and the other comes from the FC couplings . The heavier generations are naturally expected to have larger FC couplings. Then the most main contributions generated by the FC couplings to a_μ arise from the FC coupling $Z'\mu\bar{\tau}$. The relevant Feynman diagrams are shown in Fig.2. Using eq.(3), we can estimate the corrections of Z' to a_μ, which are given by:

$$
\delta a_\mu^{\mu\mu} = \frac{\alpha_e^2}{12\pi c_W^4}\frac{m_\mu^2}{K_1 M_{Z'}^2} + 0(\frac{m_\mu^2}{M_{Z'}^2}), \quad \delta a_\mu^{\mu\tau} = \frac{2\alpha_e V_{23}^2}{\pi c_W^2}\frac{m_\mu m_\tau}{M_{Z'}^2} + 0(\frac{m_\mu^2}{M_{Z'}^2}, \frac{m_\tau^2}{M_{Z'}^2}).
\tag{6}
$$

Using the relation $K_1 M_{Z'}^2 \geq \frac{\sqrt{5}\alpha_e s_W^2 M_W^2 A}{8c_W^4 \sqrt{B_\tau^{exp}(\mu\rightarrow3e)}}$[12], we can estimate the value of $\delta a_\mu^{\mu\mu}$, we find $\delta a_\mu^{\mu\mu} < 1 \times 10^{-14}$, which is at least smaller than the deviation δa_μ reported from BNL by five orders of magnitude. Thus, we can safely neglect the contributions from the $Z'\mu\bar{\mu}$ coupling. However, the numerical value of $\delta a_\mu^{\mu\tau}$ is dependent on the value of $M_{Z'}$, so we can write $\delta a_\mu^{Z'} \simeq \delta a_\mu^{\mu\tau}$ and we have:

$$
\delta a_\mu^{Z'} \simeq \frac{2\alpha_e V_{23}^2}{\pi c_W^2}\frac{m_\mu m_\tau}{M_{Z'}^2}.
\tag{7}
$$

According the idea of TC2 models, the lepton masses are dynamically generated by ETC interactions. Then the ETC gauge boson x_μ which is responsible for the mass m_μ should have contributions to the anomalous magnetic moment a_μ via the couplings given in eq.(4). The Feynman diagram is similarly to Fig.2a, only with x_μ replacing Z', and the technifermion line replacing the internal muon line. Then the contributions of x_μ to the anomalous magnetic moment a_μ can be written as[2]:

$$\delta a_\mu^{ETC} \simeq \frac{2(2-\gamma)}{2+\gamma} \frac{m_\mu^2}{M_{x_\mu}^2}. \tag{8}$$

The anomalous dimension γ is in the range of $0-2$. In our calculation, we take γ as a free parameter. Thus, for TC2 models, the total correction to a_μ can be written as:

$$\delta a_\mu \simeq \delta a_\mu^{Z'} + \delta a_\mu^{ETC} \simeq \frac{2\alpha_e V_{23}^2}{\pi c_W^2} \frac{m_\mu m_\tau}{M_{Z'}^2} + \frac{2(2-\gamma)}{2+\gamma} \frac{m_\mu^2}{M_{x_\mu}^2}. \tag{9}$$

In Fig.3, we plot the correction δa_μ as a function of the mass M_{x_μ} for $\gamma = 1$ and three values of $M_{Z'}$. One can find δa_μ is not sensitive to the value of $M_{Z'}$. As long as $1.1TeV \leq M_{x_\mu} \leq 2.2TeV$, TC2 models could explain the 2.6 standard deviation of the muon anomalous magnetic moment over its SM prediction.

To see the effects of the anomalous dimension γ on the muon anomalous magnetic moment a_μ, we plot δa_μ as a function of the parameter γ in Fig.4 for $M_{Z'} = 2TeV$ and three values of M_{x_μ}. From Fig.4 we can see that the recent experimental value of a_μ give severe bounds on the parameters of TC2 models. For $\gamma = 2$, the contributions of the ETC gauge boson x_μ to the muon anomalous magnetic moment a_μ are zero and the value of δa_μ generated by the topcolor gauge boson Z' is only 1.5×10^{-10} for $M_{Z'} = 2TeV$. In this case, if we want that TC2 models could explain the deviation $\delta a_\mu = a_\mu^{exp} - a_\mu^{SM}$, there must be $M_{Z'} \approx 550GeV$ which is not consistent with other experimental bounds on the Z' mass[12]. For $\gamma = 0$, the contributions of the ETC gauge boson x_μ are largely enhanced compared that of $\gamma = 2$. The Z' mass $M_{Z'}$ can be larger than $1.5TeV$, which is consistent with the limits on $M_{Z'}$ from other experiments. However, the mass of the ETC gauge boson x_μ must be in the range $1.9TeV \leq M_{x_\mu} \leq 3.3TeV$.

TC2 models predict a number of new particles, including PGBs (technipions, top-pions), ETC gauge bosons and extra $U(1)$ gauge boson Z'. All of these new particles

have contributions to the anomalous magnetic moment a_μ. We find the contributions of PGBs are very small which can be safely ignored. If we demand that the mass of topcolor gauge boson Z' is consistent with other experimental constrains, its contributions are smaller than that of the ETC gauge boson x_μ. As long as the muon mass is generated by the ETC interactions or other strong interactions , TC2 models could explain the deviation $\delta a_\mu = a_\mu^{exp} - a_\mu^{SM}$ for reasonable values of the parameters.

Figure captions

Fig.1: Feynman diagrams for the contributions of PGB's in TC2 models.

Fig.2: Feynman diagrams for the contributions of extra gauge boson Z' (a) $Z\mu\bar{\mu}$ coupling; (b) $Z\mu\bar{\tau}$ coupling.

Fig.3: The total correction δa_μ as a function of the mass M_{x_μ} for $\gamma = 1$, and $M_{Z'} = 0.5TeV$ (solid line), $M_{Z'} = 1TeV$ (dotted line) and $M_{Z'} = 2.5TeV$ (dashed line).

Fig.4: The total correction δa_μ in TC2 models as a function of the parameter γ for $M_{Z'} = 2TeV$, and $M_{x_\mu} = 1TeV$ (solid line), $M_{x_\mu} = 3TeV$ (dotted line) and $M_{x_\mu} = 5TeV$ (dashed line).

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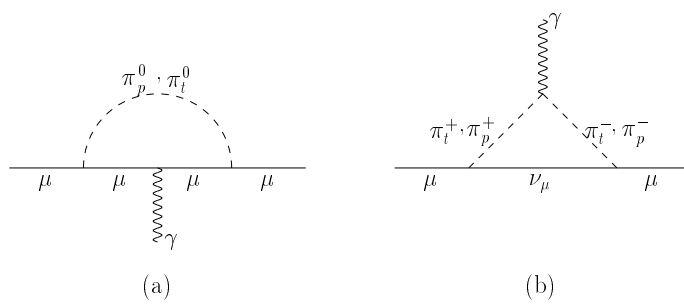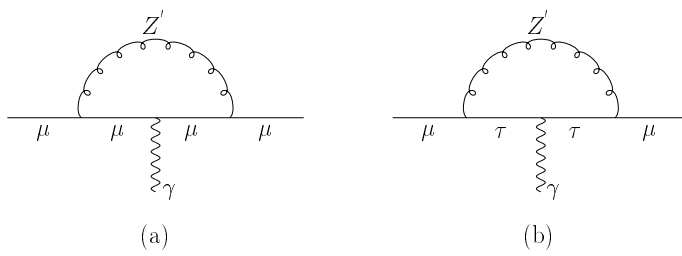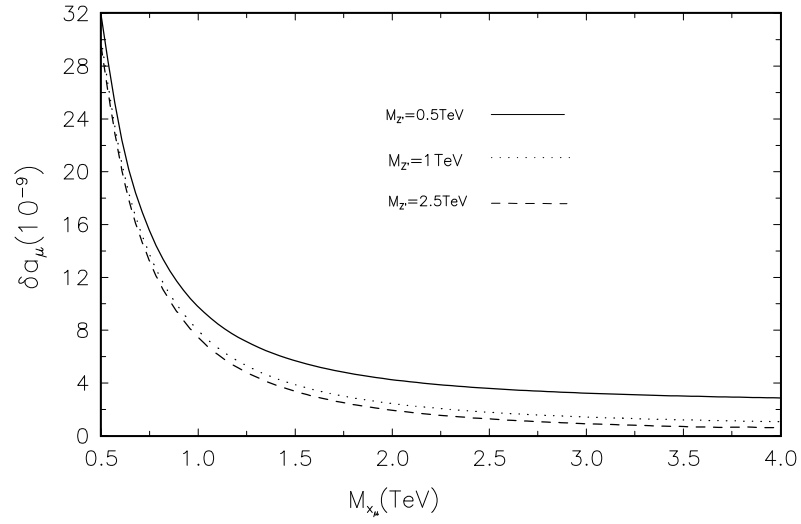[12] Chongxing Yue, Guoli Liu and Jiantao Li, *Phys. Lett.B***496** (2000)89.
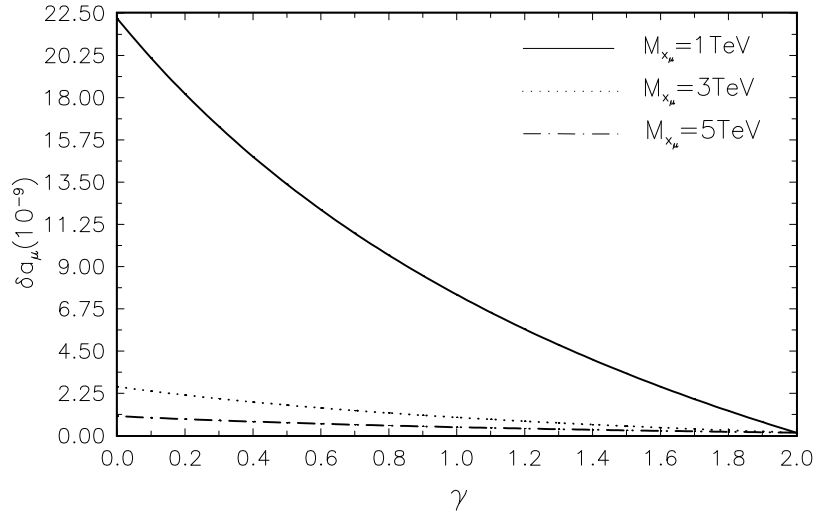
Fig.1

Fig.2

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